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Three-way cross designs for test lines vs. control line comparisons

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Abstract
The selection of appropriate parental lines and crossing plans are the most important keys for a successful breeding programme. Three-way crosses are intermediate between single and double cross hybrids and hence are economical with respect to uniformity, yield, stability and the relative simplicity of selecting and testing. When a number of test lines are to be compared to a control line, the main interest of the experimenter centers around comparing test vs. control lines with a high precision. For such situations, two series of designs have been developed using partially balanced incomplete block designs and their association schemes. In the designs obtained, three-way crosses are arranged in incomplete blocks and hence heterogeneity in the experimental field can be controlled in blocks of smaller sizes. It is found that the proposed designs make test vs. control lines comparisons with a higher precision.
Key words: Triangular association scheme, combining abilities, partially balanced incomplete block designs

Introduction

Crossbreeding has been a major tool for the development of present day commercial breeds. The most common designs used by a geneticist to study genetic parameters and their interpretations are diallele (single cross), triallele (three-way cross) and quadriallele (double cross). There are many cases of plants like maize or corn where three-way and four-way crosses are the commonly used techniques of producing commercial hybrids. Three-way crosses are more useful for plant breeding trials as these are intermediate between single and double cross hybrids with respect to uniformity, yield, stability and the relative simplicity of selecting and testing [Weatherspoon, 1970]. As the number of lines increases, the number of crosses in a complete three-way cross plan increases manifold and becomes unmanageably large for the investigator to handle. This situation lies in taking a sample of complete triallele crosses, known as partial triallele crosses. Sample is to be drawn in a systematic manner such that there is minimum loss of information regarding combining ability effects. Suitable mating designs are to be chosen carefully as the selection of an efficient crossing plan is as equally important as the selection of appropriate parental lines in a successful breeding programme.

The concept of partial triallele crosses (PTC) was introduced and the relationship between incomplete block designs and PTC is utilized for the purpose of constructing and analyzing appropriate plans by Hinkelmann (1965). Methods of construction of PTC using Trojan square design, generalized incomplete Trojan type designs and mutually orthogonal Latin squares have been obtained by several authors [Dharmalingam (2002), Varghese and Jaggi (2011) and Sharma *et al*. (2012)].

In comparative experiments, there may be an established line which may be regarded as a control line and the purpose of the trial may be to compare some new lines with the control line to find out the lines worth further study rather than to compare within new lines to find the best genotype. In this instance the greatest economy is obtained if the control is highly replicated than the new lines under test.

Diallele crosses for comparing a control line with test lines were given and designs that estimate control vs. test comparisons with a minimum variance were listed by Choi *et al*. (2004). A-optimality of diallele cross experiments for comparing two or three test lines with a control line were studied by Hsu and Ting (2005). Some classes of partial diallele cross designs for test vs. control comparisons are obtained in Srivastava *et al*. (2013). In this study, two series of partial three-way cross designs that are suitable for comparing several test lines with a control line have been obtained.

Materials and methods

Three-way cross: Definition and Model

Let there be n lines. A set of matings is said to be a PTC if it satisfies the following conditions [Hinkelmannn, 1965]:

(i) Each line occurs exactly r_H times as half-parent and r_F times as full-parent.

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(ii) Each cross $(i\times j)\times k$ occurs either once or not at all.

Condition (ii) doesn't exclude the simultaneous occurrence of $(i\times j)\times k$, $(i\times k)\times j$ and $(j\times k)\times i$. To ensure the structural symmetric property (SSP) of the PTC, all the above mentioned three types of crosses are to be included in the plan. The total number of crosses is nr_F. Since each line is equally often represented as half-parent it follows immediately that $r_H = 2r_F$.

One can obtain $N = n(n-1)(n-2)/2$ three-way crosses from n inbred lines. Consider three-way crosses (ignoring reciprocal effects) of the form $i \times j \times k$ ($i, j, k = 1,2,\ldots,n$ and $i \neq j \neq k$) arranged in b blocks of size k and each cross replicated r times. The model for mating experiments can be expressed in the form

$$y_{lm} = \mu + \tau_{(ijk)l} + \beta_m + e_{lm} \qquad (1)$$

where y_{lm} is the response from the l^{th} cross ($l = 1,2,\ldots,N$) belonging to the m^{th} ($m = 1,2,\ldots,b$) block, μ is the grand mean, $\tau_{(ijk)l}$ the effect of the l^{th} cross and e_{lm} is *i.i.d* following a normal distribution with 0 mean and constant variance σ^2.

Model (1) can be rewritten in matrix notations as

$$\boldsymbol{y}=\mu\mathbf{1}+\boldsymbol{\Delta}'\boldsymbol{\tau}+\boldsymbol{D}'\boldsymbol{\beta}+\boldsymbol{e} \qquad (2)$$

where $\boldsymbol{y}$ is a $Nr \times 1$ vector of responses, $\mathbf{1}$ is a $Nr \times 1$ vector of ones, $\boldsymbol{\Delta}'$ is a $Nr \times N$ incidence matrix of response versus crosses, $\boldsymbol{\tau}$ is a $N \times 1$ vector of cross effects and $\boldsymbol{e}$ is a $Nr \times 1$ vector of errors. Now, the design matrix $\boldsymbol{X}_{Nr \times (N+1)}$ can be partitioned into ($\boldsymbol{X}_1$) and ($\boldsymbol{X}_2$) with respect to parameters of interest and the nuisance parameters.

Some basic concepts of *m-class association scheme, partially balanced incomplete block (PBIB) design, , group divisible, triangular and cyclic association schemes* is desirable in understanding the methods of constructions explained in the succeeding sections. For more details, one may refer to Dey (1986).

Method 1: Partial three-way cross plans obtained using triangular association scheme

Let the number of lines n be of the form $v(v - 1)/2$, where $v \geq 5$. Consider an arrangement of n lines in a triangular association scheme. The empty diagonal positions are replaced by the control line $n + 1$. Making all possible distinct three-way crosses of type $(i\times j)\times k$ either within each row or column of this arrangement and ensuring the SSP of the plan, we get a partial three-way cross plan where the test lines follow a triangular association scheme. Parameters of this class of designs are as, $n = v(v - 1)/2$, $N = v^2(v - 1)(v - 2)/2$, $b = v$ and $k = v(v - 1)(v-2)/2$.

$(1/\sigma^2)$ times the average variances were computed for test vs. test and test vs. control lines of half parents ($\bar{V}_{h_{ii'}}$ and $\bar{V}_{h_{ii_0}}$) as well as full parents ($\bar{V}_{g_{ii'}}$ and $\bar{V}_{g_{ii_0}}$), where, $i \neq i' = 1,2,\ldots,n$ and $i_0 = n+1$.

The parameters of the designs obtained using triangular association scheme have been listed for $n \leq 25$ in Table 1.

Example 1: Let v=5, then the number of test lines will be 10. An arrangement of 10 test lines in a triangular association scheme is as

*	1	2	3	4
1	*	5	6	7
2	5	*	8	9
3	6	8	*	10
4	7	9	10	*

Replacing the empty diagonal positions with a control line, denoted by 11 we get the following arrangement:

11	1	2	3	4
1	**11**	5	6	7
2	5	**11**	8	9
3	6	8	**11**	10
4	7	9	10	**11**

Making all possible distinct three-way crosses within each row of this arrangement, the following partial three-way cross plan where the test lines following a triangular association scheme is obtained:

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Block I	Block II	Block III	Block IV	Block V
(11×1)×2	(1×11)×5	(2×5)×11	(3×6)×8	(4×7)×9
(1×2)×3	(11×5)×6	(5×11)×8	(6×8)×11	(7×9)×10
(2×3)×4	(5×6)×7	(11×8)×9	(8×11)×10	(9×10)×11
(3×4)×11	(6×7)×1	(8×9)×2	(11×10)×3	(10×11)×4
(4×11)×1	(7×1)×11	(9×2)×5	(10×3)×6	(11×4)×7
(11×1)×3	(1×11)×6	(2×5)×8	(3×6)×11	(4×7)×10
(1×2)×4	(11×5)×7	(5×11)×9	(6×8)×10	(7×9)×11
(2×3)×11	(5×6)×1	(11×8)×2	(8×11)×3	(9×10)×4
(3×4)×1	(6×7)×11	(8×9)×5	(11×10)×6	(10×11)×7
(4×11)×2	(7×1)×5	(9×2)×11	(10×3)×8	(11×4)×9
(11×2)×1	(1×5)×11	(2×11)×5	(3×8)×6	(4×9)×7
(1×3)×2	(11×6)×5	(5×8)×11	(6×11)×8	(7×10)×9
(2×4)×3	(5×7)×6	(11×9)×8	(8×10)×11	(9×11)×10
(3×11)×4	(6×1)×7	(8×2)×9	(11×3)×10	(10×4)×11
(4×1)×11	(7×11)×1	(9×5)×2	(10×6)×3	(11×7)×4
(11×3)×1	(1×6)×11	(2×5)×8	(3×11)×6	(4×10)×7
(1×4)×2	(11×7)×5	(5×9)×11	(6×10)×8	(7×11)×9
(2×11)×3	(5×1)×6	(11×2)×8	(8×3)×11	(9×4)×10
(3×1)×4	(6×11)×7	(8×5)×9	(11×6)×10	(10×7)×11
(4×2)×11	(7×5)×1	(9×11)×2	(10×8)×3	(11×9)×4
(1×2)×11	(11×5)×1	(5×11)×2	(6×8)×3	(7×9)×4
(2×3)×1	(5×6)×11	(11×8)×5	(8×11)×6	(9×10)×7
(3×4)×2	(6×7)×5	(8×9)×11	(11×10)×8	(10×11)×9
(4×11)×3	(7×1)×6	(9×2)×8	(10×3)×11	(11×4)×10
(11×1)×4	(1×11)×7	(2×5)×9	(3×6)×10	(4×7)×11
(3×1)×11	(6×11)×1	(8×5)×2	(11×6)×3	(10×7)×4
(4×2)×1	(7×5)×11	(9×11)×5	(10×8)×6	(11×9)×7
(11×3)×2	(1×6)×5	(2×8)×11	(3×11)×8	(4×10)×9
(1×4)×3	(11×7)×6	(5×9)×8	(6×10)×11	(7×11)×10
(2×11)×4	(5×1)×7	(11×2)×9	(8×3)×10	(9×4)×11

The parameters of this design are: total number of crosses (N) = 150, number of blocks (b) = 5, block size (k) = 30, and degree of fractionation (f) = 10/33. Now, ($1/\sigma^2$) times average variances were computed for test vs. test and test vs. control lines of half parents as $\bar{V}_{h_{ii'}} = 0.6349$ and $\bar{V}_{h_{ii_0}} = 0.4191$ as well as for full parents as $\bar{V}_{g_{ii'}} = 0.2381$ and $\bar{V}_{g_{ii_0}} = 0.1571$ to make a comparative study, where, $i \neq i' = 1,2,...,10$ and $i_0 = 11$.

Method 2: Partial three-way cross plans obtained using PBIB designs

Consider any PBIB design having small block size. Let the blocks of the design are arranged in b^* rows having block size ($k^* > 2$) made up of n test lines. Augment a control line (say, $n + 1$) to each row. Making all possible three-way crosses within each block, we can obtain a partial three-way cross plan. For this class of designs, $N = b^*k^*(k^* - 1)(k^* - 2)/6$. The parameters of the designs obtained using various PBIB designs (group divisible, triangular, cyclic) have been listed for $n \leq 25$ in Table 2.

Example 2: Partial three-way cross plans using cyclic design

Consider the blocks (arranged in rows) of a cyclic design for n = 5 and augment a control treatment 6 to each block as given below:

1, 2, 4	1, 2, 4, 6
2, 3, 5	2, 3, 5, 6
3, 4, 1	3, 4, 1, 6
4, 5, 2	4, 5, 2, 6
5, 1, 3	5, 1, 3, 6

The following partial three-way cross plan can be obtained by taking all the possible three-way crosses within each row:

Block I	Block II	Block III	Block IV	Block V
(1×2)×4	(2×3)×5	(3×4)×1	(4×5)×2	(5×1)×3
(1×2)×6	(2×3)×6	(3×4)×6	(4×5)×6	(5×1)×6
(1×4)×6	(2×5)×6	(3×1)×6	(4×2)×6	(5×3)×6
(2×4)×6	(3×5)×6	(4×1)×6	(5×2)×6	(1×3)×6

Crosses of types (i×k)×j and (j×k)×i are to be considered along with each cross (i×j)×k in each block to possess SSP. The parameters of the design

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are total number of crosses (N) = 126, number of blocks (b) = 6, block size (k) = 21, and degree of fractionation (f) = 6/8. ($1/\sigma^2$) average variances were computed for test vs. test and test vs. control lines of half parents as $\bar{V}_{h_{ii'}} = 0.2116$ and $\bar{V}_{h_{ii_0}} = 0.1300$ as well as for full parents as $\bar{V}_{g_{ii'}} = 0.0794$ and $\bar{V}_{g_{ii_0}} = 0.0524$, where $i \neq i' = 1,2,...,10$ and $i_0 = 11$.

Example 3: Partial three-way cross plans using a triangular design

Consider n = 10. A triangular PBIB design for 10 symbols and control (denoted 11) augmented blocks are given below:

1, 2, 5	1, 2, 5, 11
1, 3, 6	1, 3, 6, 11
1, 4, 7	1, 4, 7, 11
2, 3, 8	2, 3, 8, 11
2, 4, 9	2, 4, 9, 11
3, 4, 10	3, 4, 10, 11
5, 6, 8	5, 6, 8, 11
5, 7, 9	5, 7, 9, 11
6, 7, 10	6, 7, 10, 11
8, 9, 10	8, 9, 10,11

A partial three-way cross plan for test vs. control comparison obtained by taking all possible distinct three-way crosses within each block of the above design is given below:

Block I	Block II	Block III	Block IV	Block V
(1×2)×5	(1×3)×6	(1×4)×7	(2×3)×8	(2×4)×9
(1×2)×11	(1×3)×11	(1×4)×11	(2×3)×11	(2×4)×11
(1×5)×11	(1×6)×11	(1×7)×11	(2×8)×11	(2×9)×11
(2×5)×11	(3×6)×11	(4×7)×11	(3×8)×11	(4×9)×11

Block VI	Block VII	Block VIII	Block IX	Block X
(3×4)×10	(5×6)×8	(5×7)×9	(6×7)×10	(8×9)×10
(3×4)×11	(5×6)×11	(5×7)×11	(6×7)×11	(8×9)×11
(3×10)×11	(5×8)×11	(5×9)×11	(6×10)×11	(8×10)×11
(4×10)×11	(6×8)×11	(7×9)×11	(7×10)×11	(9×10)×11

Again, crosses of types (i×k)×j and (j×k)×i are to be considered along with each cross (i×j)×k in each block to possess SSP. The parameters of the design

are total number of crosses (N) = 126, number of blocks (b) = 6, block size (k) = 21, and degree of fractionation (f) = 6/8. ($1/\sigma^2$) average variances were computed for test vs. test and test vs. control lines of half parents as $\bar{V}_{h_{ii'}} = 0.2116$ and $\bar{V}_{h_{ii_0}} = 0.1300$ as well as for full parents as $\bar{V}_{g_{ii'}} = 0.0794$ and $\bar{V}_{g_{ii_0}} = 0.0524$, where $i \neq i' = 1,2,...,10$ and $i_0 = 11$.

Results and Discussion

In this paper, two different methods have been discussed for constructing PTC plans that are partially variance balanced as these methods are derived from association schemes and partially balanced incomplete block designs. The proposed designs are available for almost every parametric combination and can be easily constructed. The variance of contrasts pertaining to estimated general combining ability effects of full parents as well as half parents are computed for test vs. test lines and test vs. control lines and it was found that test vs. control lines comparisons are made with more precision. For test vs. test line comparisons, contrasts pertaining to general combining ability effects of full parents as well as half parents are estimated with two types of variances. Through the suggested methods, breeders can obtain a small and efficient three-way cross plans without vast knowledge in statistics. By using these designs, breeders can optimize the resource utilization and at the same time reduce the heterogeneity in the experimental field. Combining ability effects of full parents as well as half parents are computed for test vs. test lines and test vs. control lines and it was found that test vs. control lines

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comparisons are made with more precision. For test vs. test line comparisons, contrasts pertaining to general combining ability effects of full parents as well as half parents are estimated with two types of variances. Through the suggested methods, breeders can obtain a small and efficient three-way cross plans without vast knowledge in statistics. By using these designs, breeders can optimize the resource utilization and at the same time reduce the heterogeneity in the experimental field.

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Table 1. Partial three-way cross plans for test vs. control comparisons using triangular association scheme

Sl. No.	$n' = n+1$	b	k	N	f	$V_{h_{ii_0}}$	$V_{h_{ii'}}$	$V_{g_{ii_0}}$	$V_{g_{ii'}}$
1	11	5	30	150	10/33	0.4191	0.6349	0.1571	0.2381
2	16	6	60	360	3/14	0.2083	0.3274	0.0833	0.1310
3	22	7	105	735	49/308	0.1238	0.2000	0.0516	0.0833

Table 2. Partial three-way cross plans for test vs. control line comparisons using various PBIB designs

Sl. No.	$n' = n+1$	B	k	N	f	$V_{h_{ii_0}}$	$V_{h_{ii'}}$	$V_{g_{ii_0}}$	$V_{g_{ii'}}$	PBIB Design Used
1	11	10	12	120	0.24	0.7727	0.2727	0.2576	0.4242	triangular
2	16	20	12	240	0.14	0.5571	0.9800	0.1857	0.3266	triangular
3	22	35	12	420	0.09	0.4352	0.7941	0.1450	0.2647	triangular
4	6	5	12	60	1	0.8624	1.1559	0.2874	0.3853	cyclic
5	14	13	12	156	0.14	0.7586	1.3103	0.2528	0.4367	cyclic
6	11	4	12	48	0.10	0.3864	0.6363	0.1287	0.2121	R 69*
7	13	20	12	240	0.28	3.0000	5.4545	1.0000	1.8181	R 76*
8	7	4	12	48	0.80	1.2500	1.8	0.4166	0.6000	SR 19*

* design number in Clatworthy (1973) tables where R stands for regular group divisible design and SR stands for semi-regular group divisible design.